Sefydliad y Gwyddorau Mathemategol a Ffisegol

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Fformwlâu Mathemategol a Chysonion Sylfaenol

Mathematical Formulae and Fundamental Constants

Differiadau

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
k, cysonyn	0
x^n, unrhyw gysonyn n	nx^{n-1}
e^x	e^x
$\ln x = \log_e x$	$\frac{1}{x}$
sinx	cosx
cosx	-sinx
tanx	$\sec^2 x$
cotx	$-\text{cosec}^2 x$
sinhx	coshx
coshx	sinhx

Integriadau

f(x)	$\int f(x)dx = F(x) + c$
k, cysonyn	$kx + c$
x^n, $(n \neq -1)$	$\frac{1}{n+1} x^{n+1}$
e^x	$e^x + c$
$\frac{1}{x}$	$\ln x + c \quad (x > 0)$
	$\ln(-x) + c \quad (x < 0)$
$\log_e x$	$x\log_e x - x$
sinx	$-\cos x + c$
cosx	$\sin x + c$
tanx	$\ln(\sec x) + c$
cotx	$\ln(\sin x) + c$
sinhx	$\cosh x + c$
coshx	$\sinh x + c$
$\frac{1}{\sqrt{(1-x^2)}}$	$\sin^{-1} x + c$
$\frac{1}{1+x^2}$	$\tan^{-1} x + c$

Deddfau Indecsau

$a^m a^n = a^{(m+n)}$
$\frac{a^m}{a^n} = a^{(m-n)}$
$(a^m)^n = a^{mn}$
$a^0 = 1$
$a^{-m} = \frac{1}{a^m}$
$a^{1/n} = \sqrt[n]{a}$

Newidynnau Cymhlyg

z = newidyn cymhlyg,
x, y = newidynnau real
r = osgled (real)
θ = cydwedd (real)
|z| = modwlws z
arg z = arg z
z* = cyfiau cymhlyg z
n = cyfanrif

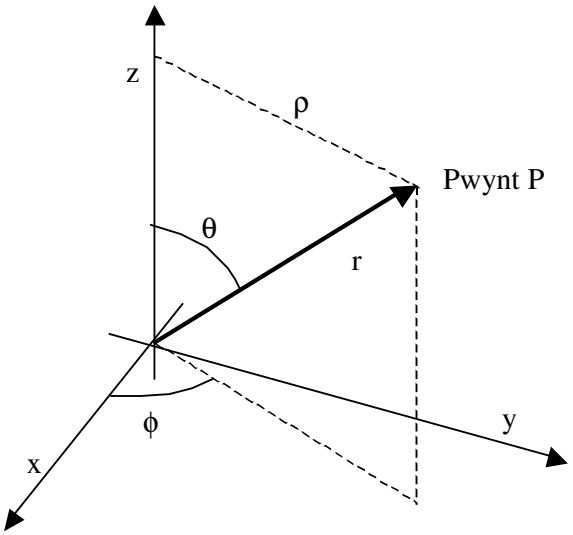
Ffurf cartesaidd	$z = x + jy$
Ffurf polar	$z = re^{j\theta} = r(\cos\theta + j\sin\theta)$
Modwlws	$\|z\| = r = (x^2 + y^2)^{1/2}$
Arg	$\theta = \arg z = \arctan(y/x)$
Cyfiau cymhlyg	$z^* = x - jy = re^{-j\theta}$
Theorem de Moivre	$(\cos\theta + j\sin\theta)^n = \cos n\theta + j\sin n\theta$

Geometreg

Ar gyfer cylch o radiws r a diamedr d,
Cylchedd = $2\pi r = \pi d$
Arwynebedd = $\pi r^2 = \pi d^2/4$

Ar gyfer sffêr o radiws r,
Arwynebedd arwyneb = $4\pi r^2$
Cyfaint = $\frac{4}{3}\pi r^3$
Ar gyfer silindr o radiws r ac uchder h,
Arwynebedd arwyneb = $2\pi rh$
Cyfaint = $\pi r^2 h$

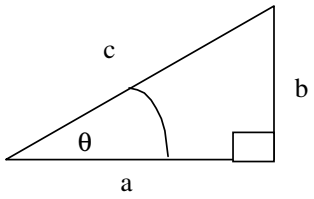
Systemau cyfesurynnau tri dimensiwn cyffredin



Cartesaidd	**Silindrog**	**Sfferig**
X	ρcosϕ	rsinθcosϕ
y	ρsinϕ	rsinθsinϕ
z	Z	Rcosθ

Trigonometreg

$360^0 = 2\pi$ radianau



$\sin\theta = b/c$ $\cos\theta = a/c$ $\tan\theta = b/a$

$\sin 45^0 = 1/\sqrt{2}$ $\cos 45^0 = 1/\sqrt{2}$ $\tan 45^0 = 1$

$\sin 30^0 = 1/2$ $\cos 30^0 = \sqrt{3}/2$ $\tan 30^0 = 1/\sqrt{3}$

$\sin 60^0 = \sqrt{3}/2$ $\cos 60^0 = 1/2$ $\tan 60^0 = \sqrt{3}$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
$\sin^2 A + \cos^2 A = 1$
$\cos^2 A - \sin^2 A = \cos 2A$
$\sin 2A = 2\sin A\cos A$

$\cos(-A) = \cos A$
$\sin(-A) = -\sin A$
$\sin(\pi + A) = -\sin A$
$\sin(\pi - A) = \sin A$
$\sin(\pi/2 + A) = \cos A$
$\sin(\pi/2 - A) = \cos A$
$\cos(\pi + A) = -\cos A$
$\cos(\pi - A) = -\cos A$
$\cos(\pi/2 + A) = -\sin A$
$\cos(\pi/2 - A) = \sin A$

Matricsau a Determinantau

Mae gan y matrics 2 x 2 $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ ddeterminant

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

ac mae ganddo wrthdro

$$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

cyn belled bo $ad - bc \neq 0$

Mae gan y matrics 3 x 3 $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ ddeterminant

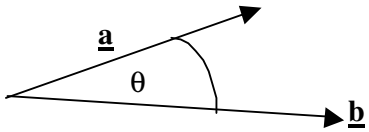
$|A| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$

Fectorau

Os yw $\underline{\mathbf{r}} = x\underline{\mathbf{i}} + y\underline{\mathbf{j}} + z\underline{\mathbf{k}}$ yna $|\underline{\mathbf{r}}| = \sqrt{x^2 + y^2 + z^2}$

$\underline{\mathbf{a}}.\underline{\mathbf{b}} = |\underline{\mathbf{a}}||\underline{\mathbf{b}}|\cos\theta$

$\underline{\mathbf{a}} \times \underline{\mathbf{b}} = |\underline{\mathbf{a}}||\underline{\mathbf{b}}|\sin\theta\ \hat{\mathbf{e}}$ lle bo $\hat{\mathbf{e}}$ yn fector uned sy'n berpendicwlar i'r plân sy'n cynnwys $\underline{\mathbf{a}}$ a $\underline{\mathbf{b}}$



Ar gyfer maes sgalar f, $\nabla f = \frac{\partial f}{\partial x}\underline{\mathbf{i}} + \frac{\partial f}{\partial y}\underline{\mathbf{j}} + \frac{\partial f}{\partial z}\underline{\mathbf{k}}$ lle bo $\underline{\mathbf{i}}$, $\underline{\mathbf{j}}$, $\underline{\mathbf{k}}$ yn fectorau uned ar hyd x,y,z

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$ cyfesurynnau petryalog

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(r\frac{\partial f}{d\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$ cyfesurynnau silindrog

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{dr}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$ cyfesurynnau sfferig polar

Ar gyfer maes fector $\underline{\mathbf{A}}$, gyda chydrannau A_x, A_y, A_z ar hyd x, y, z

$$\nabla . \underline{\mathbf{A}} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \underline{\mathbf{A}} = \begin{vmatrix} \underline{\mathbf{i}} & \underline{\mathbf{j}} & \underline{\mathbf{k}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla(fg) = f\nabla g + g\nabla f$$
$$\nabla.(f\underline{A}) = f\nabla.\underline{\mathbf{A}} + \underline{\mathbf{A}}.\nabla f$$
$$\nabla x(f\underline{\mathbf{A}}) = f\nabla x\underline{\mathbf{A}} + (\nabla f)x\underline{\mathbf{A}}$$
$$\nabla(\underline{\mathbf{A}}.\underline{\mathbf{B}}) = \underline{\mathbf{A}}x(\nabla x\underline{\mathbf{B}}) + (\underline{\mathbf{A}}.\nabla)\underline{\mathbf{B}} + \underline{\mathbf{B}}x(\nabla x\underline{\mathbf{A}}) + (\underline{\mathbf{B}}.\nabla)\underline{\mathbf{A}}$$
$$\nabla.(\underline{\mathbf{A}}x\underline{\mathbf{B}}) = \underline{\mathbf{B}}.(\nabla x\underline{\mathbf{A}}) - \underline{\mathbf{A}}.(\nabla x\underline{\mathbf{B}})$$
$$\nabla x(\underline{\mathbf{A}}x\underline{\mathbf{B}}) = \underline{\mathbf{A}}(\nabla.\underline{\mathbf{B}}) - \underline{\mathbf{B}}(\nabla.\underline{\mathbf{A}}) + (\underline{\mathbf{B}}.\nabla)\underline{\mathbf{A}} - (\underline{\mathbf{A}}.\nabla)\underline{\mathbf{B}}$$
$$\nabla.(\nabla f) = \nabla^2 f$$
$$\nabla x(\nabla f) = 0$$
$$\nabla.(\nabla x\underline{\mathbf{A}}) = 0$$
$$\nabla x(\nabla x\underline{\mathbf{A}}) = \nabla(\nabla.\underline{\mathbf{A}}) - \nabla^2\underline{\mathbf{A}}$$

Theorem (dargyfeiriad) Gauss: $\int_V (\nabla.\underline{\mathbf{A}})dV = \oint_S \underline{\mathbf{A}}.d\underline{\mathbf{s}}$

Theorem Stokes: $\int_S (\nabla x\underline{\mathbf{A}})d\underline{\mathbf{s}} = \oint_L \underline{\mathbf{A}}.d\underline{\boldsymbol{\ell}}$

Lle bo:
$\underline{\mathbf{A}}$ yn faes fector
dV yn gyfaint elfen
S_c yn arwyneb caeëdig
V yn gyfaint amgaeëdig
S yn arwyneb
$d\underline{\mathbf{s}}$ yn arwyneb elfen
L yn ddolen sy'n ffinio S
$d\underline{\boldsymbol{\ell}}$ yn llinell elfen

Cysonion Sylfaenol / Fundamental Constants

Mesur	Symbol	Gwerth
pi	π	3.141592
bôn logarithmau naturiol	e	2.718282
Uned Angstrom	Å	10^{-10} m
Buanedd goleuni mewn gwactod	c	3×10^{8} ms^{-1}
Cysonyn Planck	h	6.63×10^{-34} Js
Cysonyn Planck wedi'i addasu $h/2\pi$	$\hbar$	1.05×10^{-34} Js
Gwefr electronig e	e	1.6×10^{-19} C
Cyflymiad oherwydd disgyrchiant	g	9.8 m s^{-2}
Cysonyn disgyrchedd	G	6.67×10^{-11} N m^2 kg^{-2}
Permitifedd gwactod	ε_0	8.85×10^{-12} F m^{-1}
Athreiddedd gwactod	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Màs disymud electron	m_e	9.1×10^{-31} kg
Egni cywerth màs disymud electron		0.51 MeV
Màs disymud proton	M_p	1.6726×10^{-27} kg
Egni cywerth màs disymud proton		938 MeV
Màs disymud niwtron	M_n	1.6749×10^{-27} kg
Egni cywerth màs disymud niwtron		940 MeV
Moment magnetig electron	μ_e	9.28×10^{-24} J T^{-1}
Moment magnetig proton	μ_p	1.41×10^{-26} J T^{-1}
Magneton Bohr	μ_B	9.27×10^{-24} J T^{-1}
Magneton niwclear	μ_N	5.05×10^{-27} J T^{-1}
Cysonyn nwy	R	8.3 J (mol K)$^{-1}$
Rhif Avogadro	N_A	6×10^{23} mol^{-1}
Cysonyn Boltzmann	k_B	1.38×10^{-23} J K^{-1}
Cysonyn Stefan-Boltzmann	σ	5.67×10^{-8} W m^{-2} K^{-4}
Radiws Bohr	a_0	5.3×10^{-11} m
Uned màs atomig (a.m.u.)		1.66×10^{-27} kg
Cysonyn strwythur mân	α	1/137
1 atmosffêr		1.01×10^{5} Pa
1 electron folt (eV)		1.6×10^{-19} J
Màs yr Haul		2×10^{30} kg
Màs y Ddaear		6×10^{24} kg
Radiws y Ddaear		6.38×10^{6} m
Radiws yr Haul		6.96×10^{8} m
1 AU		1.5×10^{11} m
1 parsec		3.086×10^{16} m